

# Soft-Gluon Resummation for Heavy Quark Production in Charged-Current Deep Inelastic Scattering

G. Corcella <sup>1</sup> and A.D. Mitov <sup>2</sup>

<sup>1</sup> Max-Planck-Institut für Physik, Werner-Heisenberg-Institut,  
Föhringer Ring 6, D-80805 München, Germany .

<sup>2</sup> Department of Physics and Astronomy, University of Rochester,  
Rochester, NY 14627, U. S. A.

## Abstract

We study soft-gluon radiation for heavy quark production in charged-current Deep Inelastic Scattering processes. We resum large- $x$  contributions to the  $\overline{\text{MS}}$  coefficient function to next-to-leading logarithmic accuracy and present results for charm quark structure functions for several values of the ratio  $m_c/Q$ . The effect of soft resummation turns out to be visible, especially at small  $Q^2$ , and the results exhibit very little dependence on factorization and renormalization scales. The impact of our calculation on structure function measurements at NuTeV and HERA experiments is discussed.

# 1 Introduction

Lepton-hadron Deep Inelastic Scattering (DIS) is one of the most interesting processes to investigate Quantum Chromodynamics, from both theoretical and experimental point of view. In fact, DIS allows one to measure proton structure functions, and hence investigate the internal structure of hadrons. For the sake of performing accurate measurements of observables in DIS processes, precise QCD calculations are necessary.

In this paper we study heavy quark production in DIS, in particular charm quark production in charged-current (CC) events. Measurements of the charm quark structure functions in the CC regime are in fact important in order to probe the density of strange quarks and gluons in the proton.

Charged-current Deep Inelastic Scattering processes  $\nu_\mu N \rightarrow \mu X$  are currently under investigation at the NuTeV experiment at Fermilab, where neutrino and antineutrino beams collide on an fixed iron target (see, e.g., Ref. [1] for the updated results). In particular, production of two oppositely-charged muons at NuTeV [2] is mainly associated with a CC event with a charm quark in the final state.

The first generation of H1 and ZEUS experiments on electron-proton DIS at the HERA collider at DESY (HERA I) did detect charged-current events  $ep \rightarrow \nu_e X$  [3, 4], but it did not have sufficiently-high statistics to reconstruct heavy quarks in the CC regime. However, such measurements are foreseen in the current improved-luminosity run (HERA II) and at the possible future generation of HERA III experiments [5], which should also be able to investigate the proton structure functions at low  $Q^2$  values.

Structure functions are given by convolutions of short-distance, partonic coefficient functions,  $C(x, \mu_F^2)$ , usually given in the  $\overline{\text{MS}}$  factorization scheme, with long-distance parton distribution functions  $f(x, \mu_F^2)$ , whose dependence on the factorization scale  $\mu_F$  is ruled by the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations [6, 7]. The next-to-leading order (NLO)  $\overline{\text{MS}}$  coefficient functions for charm production in CC DIS present terms which get arbitrarily large once the incoming quark energy fraction  $z$  in  $qW^* \rightarrow c(g)$  approaches unity. This corresponds to soft-gluon radiation. In order to reliably predict structure functions at large  $z$ , such terms need to be resummed to all orders in the strong coupling constant  $\alpha_S$  (threshold resummation). It is the purpose of this paper to perform the resummation of the  $\overline{\text{MS}}$  coefficient function and investigate the phenomenological implications of the resummation on charm quark structure functions at NuTeV and HERA.

Soft resummation for the coefficient function in DIS was first considered in [8, 9, 10] and later implemented in [11], but in the approximation in which all participating quarks are massless. Soft-gluon resummation for heavy quark production in DIS processes

has been investigated in [12, 13], where the authors have considered neutral current interactions. Phenomenological studies at NLO on charm quark production at NuTeV and HERA have been performed in [14] and [15] respectively.

The plan of our paper is the following. In Section 2 we review the NLO result for heavy quark production in CC DIS and discuss the behaviour of the coefficient function for soft-gluon radiation. In Section 3 we present results for the soft-resummed  $\overline{\text{MS}}$  coefficient function. In particular, we shall compare the results on CC DIS with soft resummation in top quark decay [16, 17] and we shall comment on the differences with respect to the previous work on light quark production in DIS [8, 10, 11]. In Section 4 we present our predictions on charm quark structure functions for several values of  $Q^2$  which are relevant for NuTeV and HERA phenomenology. In Section 5 we shall summarize the main results of our work and discuss its possible future extensions.

## 2 Coefficient functions and structure functions at NLO

In this section we would like to review the main results on CC DIS at NLO in the strong coupling constant  $\alpha_S$ . The Born parton-level process in CC DIS reads:

$$q_1(p_1)W^*(q) \rightarrow q_2(p_2). \quad (1)$$

Process (1) receives  $\mathcal{O}(\alpha_S)$  corrections from quark-scattering

$$q_1(p_1)W^*(q) \rightarrow q_2(p_2)(g(p_g)) \quad (2)$$

and gluon-fusion process

$$g(p_g)W^*(q) \rightarrow \bar{q}_1(p_1)q_2(p_2). \quad (3)$$

Hereafter, we shall assume that  $q_1$  is light and  $q_2$  is heavy, i.e.  $p_1^2 = 0$  and  $p_2^2 = m^2$ . We also define  $Q^2 = -q^2$  and the Bjorken  $x$  as

$$x = \frac{Q^2}{2P \cdot q}, \quad (4)$$

where  $P$  is momentum of the nucleon upon which the incoming lepton scatters, i.e. a proton for  $ep$  interaction, a proton or a neutron for  $\nu N$  processes.

In the following, in order to appreciate quark mass effects, we shall be mainly interested in values of  $Q^2$  with respect to which  $m^2$  is not negligible. Nevertheless, we shall present results for both small and large values of  $m^2/Q^2$  and comment on the transition between the massive case and the massless approximation.

At small values of  $Q^2$  one should also take into account the effect of the target mass  $M$ , typically of the order of 1 GeV. This leads to the introduction of the Nachtmann variable  $\eta$ , which generalizes the Bjorken  $x$  to the case of non-negligible target mass. It satisfies the following relation:

$$\frac{1}{\eta} = \frac{1}{2x} + \sqrt{\frac{1}{4x^2} + \frac{M^2}{Q^2}}. \quad (5)$$

The coefficient function of the quark-scattering process (2) is usually expressed in terms of the variable

$$z = \frac{Q^2 + m^2}{2p_1 \cdot q}, \quad (6)$$

with  $0 \leq z \leq 1$ . Here we also introduce the energy fraction  $\xi$  of  $q_1$  with respect to the initial-state proton<sup>1</sup> and the quantity

$$\chi = \frac{\eta}{\lambda}, \quad (7)$$

where we have defined

$$\lambda = \frac{Q^2}{Q^2 + m^2}. \quad (8)$$

In the approximation of vanishing target and quark masses,  $\chi$  coincides with the Bjorken  $x$ , i.e.  $\chi \rightarrow x$ . Moreover, the relation  $z = \chi/\xi$  holds and  $x$  and  $\chi$  are related via [18]:

$$x = \frac{\lambda\chi}{1 - M^2\lambda^2\chi^2/Q^2}. \quad (9)$$

The Bjorken  $x$ , which in the massless approximation varies between 0 and 1, is now constrained in the range:

$$0 < x \leq \frac{\lambda}{1 - M^2\lambda^2/Q^2}. \quad (10)$$

The differential distributions  $d\hat{\sigma}/dz$  of the parton-level processes in Eqs. (2) and (3) are collinear divergent and their explicit expressions can be found in [19]. Such collinear singularities are usually regularized in dimensional regularization and subtracted in the  $\overline{\text{MS}}$  factorization scheme. This procedure yields the  $\overline{\text{MS}}$  coefficient functions [20].

Following the notation of [21], and referring to neutrino scattering on a fixed target, the inclusive cross section for  $\nu(\bar{\nu})$  scattering in the CC regime can be expressed as a linear combination of three structure functions  $F_1$ ,  $F_2$  and  $F_3$ :

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi(1 + Q^2/m_W^2)^2} \left\{ y^2 x F_1 + \left[ 1 - \left( 1 + \frac{Mx}{2E} \right) y \right] F_2 \pm y \left( 1 - \frac{y}{2} \right) x F_3 \right\}. \quad (11)$$

<sup>1</sup> $\xi$  can be defined, e.g., in a frame where the nucleon and the quark are collinear [18]. Given  $P = (P^{(0)}, 0, 0, P^{(3)})$  and  $p_1 = (p_1^{(0)}, 0, 0, p_1^{(3)})$ , one has:  $\xi = (p_1^{(0)} + p_1^{(3)}) / (P^{(0)} + P^{(3)})$ .

In the above equation,  $G_F$  is the Fermi constant,  $E$  is the neutrino energy in the target rest frame and  $y$  the inelasticity variable, given by:

$$y = \frac{P \cdot q}{P \cdot k}, \quad (12)$$

with  $k$  being the momentum of the incoming lepton, the neutrino in the case of Eq. (11). In Eq. (11), we have ignored structure functions  $F_4$  and  $F_5$  since their contribution to the differential cross section is proportional to powers of the lepton masses and is therefore suppressed [21]. Analogous expressions to Eq. (11) hold for positron (electron) scattering on a proton as well.

Following [20, 21, 22], one defines the ‘theoretical’ structure functions  $\mathcal{F}_i$  which are related to the  $F_i$ ’s of Eq. (11) via the following relations:

$$F_1 = \mathcal{F}_1, \quad (13)$$

$$F_2 = \frac{2x}{\lambda \rho^2} \mathcal{F}_2, \quad (14)$$

$$F_3 = \frac{2}{\rho} \mathcal{F}_3, \quad (15)$$

where we have defined:

$$\rho = \sqrt{1 + \left( \frac{2Mx}{Q} \right)^2}. \quad (16)$$

The introduction of the structure functions  $\mathcal{F}_i$  is convenient as they can be straightforwardly expressed as the convolution of parton distribution functions with  $\overline{\text{MS}}$  coefficient functions:

$$\mathcal{F}_i(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} \left[ C_i^q(\xi, \mu^2, \mu_F^2, \lambda) q_1\left(\frac{\chi}{\xi}, \mu_F^2\right) + C_i^g(\xi, \mu^2, \mu_F^2, \lambda) g\left(\frac{\chi}{\xi}, \mu_F^2\right) \right], \quad (17)$$

for  $i = 1, 2, 3$ . In Eq. (17),  $\mu$  and  $\mu_F$  are the renormalization and factorization scales;  $C_i^q$  and  $C_i^g$  are the coefficient functions for quark-scattering (2) and gluon fusion (3) processes;  $q_1(\xi, \mu_F^2)$  and  $g(\xi, \mu_F^2)$  are the initial-state light quark and gluon parton distribution functions in (2) and (3). We have chosen  $x$  as the argument of  $\mathcal{F}_i$  since later we shall present results for the structure functions in terms of  $x$ .

The coefficient functions can be evaluated perturbatively and, to  $\mathcal{O}(\alpha_S)$ , they read:

$$C_i^q(z, \mu^2, \mu_F^2, \lambda) = \delta(1 - z) + \frac{\alpha_S(\mu^2)}{2\pi} H_i^q(z, \mu_F^2, \lambda), \quad (18)$$

$$C_i^g(z, \mu^2, \mu_F^2, \lambda) = \frac{\alpha_S(\mu^2)}{2\pi} H_i^g(z, \mu_F^2, \lambda). \quad (19)$$

The expressions for the functions  $H_{1,2,3}$  with the collinear divergences subtracted in the  $\overline{\text{MS}}$  factorization scheme can be found in [20]. For consistency, in Eq. (17) one will have to use  $\overline{\text{MS}}$  parton distribution functions as well.

As we are inclusive with respect to the final-state heavy quark  $q_2$ , the quark-scattering coefficient functions are free from mass logarithms  $\ln(m^2/Q^2)$ . However, the functions  $H_i^q$  present terms which behave like  $\sim 1/(1-z)_+$  or  $\sim [\ln(1-z)/(1-z)]_+$  and are therefore enhanced once the quark energy fraction  $z$  approaches one [20]. The  $z \rightarrow 1$  limit corresponds to soft-gluon emission. In Mellin space, such a behaviour corresponds to contributions  $\sim \ln N$  or  $\sim \ln N^2$  in the limit  $N \rightarrow \infty$ , where the Mellin transform of a function  $f(z)$  is defined as:

$$f_N = \int_0^1 dz z^{N-1} f(z). \quad (20)$$

The gluon-initiated coefficient functions  $H_i^g$  in Eq. (19) are however not enhanced in the limit  $z \rightarrow 1$ , since the splitting  $g \rightarrow q\bar{q}$  is not soft divergent.

We would like to make further comments on the coefficient functions at large  $z$ . Omitting terms that are not enhanced in the soft limit for *any* value of the mass ratio  $m^2/Q^2$ , one has:

$$H_i^q(z \rightarrow 1, \mu_F^2, \lambda) = H^{\text{soft}}(z, \mu_F^2, \lambda), \quad (21)$$

where

$$\begin{aligned} H^{\text{soft}}(z, \mu_F^2, \lambda) &= 2C_F \left\{ 2 \left[ \frac{\ln(1-z)}{1-z} \right]_+ - \left[ \frac{\ln(1-\lambda z)}{1-z} \right]_+ \right. \\ &\quad \left. + \frac{1}{4} \left[ \frac{1-z}{(1-\lambda z)^2} \right]_+ + \frac{1}{(1-z)_+} \left( \ln \frac{Q^2 + m^2}{\mu_F^2} - 1 \right) \right\}, \end{aligned} \quad (22)$$

with  $C_F = 4/3$ .

One immediately sees that the behaviour of the coefficient function (22) at large  $z$  strongly depends on the value of the ratio  $m/Q$ . To illustrate this point, we consider the two extreme regimes: small- $m/Q$  limit, i.e.  $\lambda \approx 1$ , and large- $m/Q$  limit, i.e.  $\lambda \ll 1$ .

In the first case, setting  $\lambda = 1$  in (22) and performing its Mellin transform, one recovers the large- $N$  limit of the  $N$ -space  $\overline{\text{MS}}$  massless coefficient function  $C_N$  reported in [10]:

$$C_N^{\text{soft}}(\mu^2, \mu_F^2, Q^2)|_{\lambda=1} = 1 + \frac{\alpha_S(\mu^2)C_F}{\pi} \left\{ \frac{1}{2} \ln^2 N + \left[ \gamma_E + \frac{3}{4} - \ln \frac{Q^2}{\mu_F^2} \right] \ln N \right\}, \quad (23)$$

where  $\gamma_E = 0.577 \dots$  is the Euler constant.

In the second case, for  $\lambda \rightarrow 0$  and  $z \rightarrow 1$ , one has  $\lambda z = \lambda$  in (22). As a result, the term  $\sim [(1-z)/(1-\lambda z)^2]_+$  is regular in the soft limit. The following expression holds in moment space:

$$C_N^{\text{soft}}(\mu^2, \mu_F^2, Q^2, m^2)|_{\lambda \ll 1} = 1 + \frac{\alpha_S(\mu^2)C_F}{\pi} \left\{ \ln^2 N + \left[ 2\gamma_E + 1 - \ln \frac{\mathcal{M}^2}{\mu_F^2} \right] \ln N \right\}. \quad (24)$$

For later convenience, we have introduced the scale:

$$\mathcal{M}^2 = m^2 \left( 1 + \frac{Q^2}{m^2} \right)^2. \quad (25)$$

Furthermore, in evaluating the Mellin transforms, we have made use of the following relation [8]:

$$\int_0^1 dz z^{N-1} \left[ \frac{\ln^k(1-z)}{(1-z)} \right]_+ = \begin{cases} -\ln N + \mathcal{O}(1) & \text{for } k = 0, \\ 1/2 \ln^2 N + \gamma_E \ln N + \mathcal{O}(1) & \text{for } k = 1. \end{cases}$$

The different behaviour in the soft limit of the NLO  $\overline{\text{MS}}$  coefficient function for small and large values of the ratio  $m/Q$ , which can be seen by comparing Eqs.(23) and (24), will lead to different expressions of the resummed coefficient function, which will depend on the value of  $m/Q$ . This will be discussed in detail in the next section.

### 3 Soft-gluon resummation

In this section we would like to discuss the resummation of soft-gluon radiation in the quark-initiated  $\overline{\text{MS}}$  coefficient function of CC DIS.

Soft resummation in DIS can be performed along the general lines of [8]: one evaluates the amplitude of the process  $q_1(p_1)W^*(q) \rightarrow q_2(p_2)g(p_g)$  in the eikonal approximation and exponentiates the result.

However, we can think of a simpler solution to get the resummed coefficient function in massive CC DIS. We note that, in the eikonal approximation, the coefficient function for heavy quark production in charged-current DIS is related by crossing to the coefficient function of heavy-quark decay into a  $W$  and a light quark, for example top decay  $t(p_t) \rightarrow W(q)b(p_b)(g(p_g))$ . In fact, in top decay the  $b$  quark can be treated in the massless approximation  $m_b \simeq 0$ , since  $m_b^2 \ll m_t^2$  and  $m_b^2 \ll m_W^2$ .

If in top quark decay one observes the  $b$  quark, i.e. computes the differential width with respect to the  $b$ -quark energy fraction  $x_b$  [16, 17], one recovers a situation which is analogous to CC DIS. We detect a light quark, namely the  $b$  in the top decay coefficient

function and  $q_1$  in CC DIS, and are inclusive with respect to the heavy quark, the top quark in  $t$ -decay, the massive quark  $q_2$  in process (2).

The correspondence with the top-decay process implies that, for large  $N$ , the CC DIS large- $m/Q$  coefficient function (24) coincides with the top decay coefficient function computed in Refs. [16, 17] after one performs the replacements:

$$m_t \rightarrow m \quad ; \quad m_W^2 \rightarrow -Q^2. \quad (26)$$

We can therefore apply Eq. (26), along with  $p_t \rightarrow p_2$  and  $p_b \rightarrow p_1$  in the eikonal current of Ref. [17] and repeat all the steps which have led to the resummation of the top decay  $\overline{\text{MS}}$  coefficient function, which we do not report here for the sake of brevity, and resum soft contributions in CC DIS to next-to-leading logarithmic (NLL) accuracy.

As in [17], we write the resummed coefficient function as an integral over  $z$  and  $k^2 = (p_1 + p_g)^2(1 - z)$ , with  $\mu_F^2 \leq k^2 \leq \mathcal{M}^2(1 - z)^2$ , and obtain the following expression:

$$\ln \Delta_N = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \left\{ \int_{\mu_F^2}^{\mathcal{M}^2(1-z)^2} \frac{dk^2}{k^2} A[\alpha_S(k^2)] + S[\alpha_S(\mathcal{M}^2(1 - z)^2)] \right\}, \quad (27)$$

where  $\mathcal{M}$  has been introduced in Eq. (25).

We point out that the upper limit of the integration over  $k^2$  can be explicitly obtained, e.g., in the centre-of-mass frame, where the light-quark energy satisfies the relation

$$2E_1 \simeq \frac{Q^2 + m^2}{\sqrt{(1 - z)Q^2 + m^2}}. \quad (28)$$

For finite  $m$ , in the soft limit  $z \rightarrow 1$ ,  $2E_1 \simeq (Q^2 + m^2)/m$ . From  $k^2 \leq 4E_1^2(1 - z)^2$ , one gets the upper limit  $k^2 \leq \mathcal{M}(1 - z)^2$ .

The function  $A(\alpha_S)$  in Eq. (27) can be expanded as a series in  $\alpha_S$  as:

$$A(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n A^{(n)}. \quad (29)$$

The first two coefficients are mandatory in order to resum our coefficient function to NLL accuracy [8]:

$$A^{(1)} = C_F, \quad (30)$$

$$A^{(2)} = \frac{1}{2}C_F \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9}n_f \right], \quad (31)$$

where  $C_F = 4/3$ ,  $C_A = 3$  and  $n_f$  is the number of quark flavours. A numerical estimate for  $A^{(3)}$  is also known [23] (see also [11]).



The function  $S(\alpha_S)$  is characteristic of processes where a heavy quark is involved and expresses soft radiation which is not collinear enhanced.

The following expansion holds:

$$S(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n S^{(n)}, \quad (32)$$

where only the first coefficient is needed to NLL level. It reads:

$$S^{(1)} = -C_F. \quad (33)$$

The integral over  $z$  in Eq. (27) can be performed, up to NLL accuracy, by making the following replacement [8]:

$$z^{N-1} - 1 \rightarrow -\Theta \left( 1 - \frac{e^{-\gamma_E}}{N} - z \right), \quad (34)$$

where  $\Theta$  is the Heaviside step function. This leads to writing the following result for the function  $\Delta_N$ :

$$\Delta_N(\mu^2, \mu_F^2, m^2, Q^2) = \exp \left[ \ln N g^{(1)}(\ell) + g^{(2)}(\ell, \mu^2, \mu_F^2) \right], \quad (35)$$

with

$$\ell = b_0 \alpha_S(\mu^2) \ln N, \quad (36)$$

and the functions  $g^{(1)}$  and  $g^{(2)}$  given by

$$\begin{aligned} g^{(1)}(\ell) &= \frac{A^{(1)}}{2\pi b_0 \ell} [2\ell + (1 - 2\ell) \ln(1 - 2\ell)], \\ g^{(2)}(\ell, \mu^2, \mu_F^2) &= \frac{A^{(1)}}{2\pi b_0} \left[ \ln \frac{\mathcal{M}^2}{\mu_F^2} - 2\gamma_E \right] \ln(1 - 2\ell) \\ &+ \frac{A^{(1)} b_1}{4\pi b_0^3} [4\ell + 2 \ln(1 - 2\ell) + \ln^2(1 - 2\ell)] \\ &- \frac{1}{2\pi b_0} [2\ell + \ln(1 - 2\ell)] \left( \frac{A^{(2)}}{\pi b_0} + A^{(1)} \ln \frac{\mu^2}{\mu_F^2} \right) \\ &+ \frac{S^{(1)}}{2\pi b_0} \ln(1 - 2\ell). \end{aligned} \quad (37)$$

$$(38)$$

In Eqs. (36-38),  $b_0$  and  $b_1$  are the first two coefficients of the QCD  $\beta$  function:

$$b_0 = \frac{33 - n_f}{12\pi}, \quad b_1 = \frac{153 - 19n_f}{24\pi^2}. \quad (39)$$

In Eq. (35) the term  $\ln N g^{(1)}(\ell)$  accounts for the resummation of leading logarithms (LL)  $\alpha_S^n \ln^{n+1} N$  in the Sudakov exponent, while function  $g^{(2)}(\ell, \mu^2, \mu_F^2)$  resums NLL

terms  $\alpha_S^n \ln^n N$ . One can also check that the  $\mathcal{O}(\alpha_S)$  expansion of Eq.(35) reproduces the large- $N$  coefficient function in Eq. (24).

Furthermore, we follow [17, 24] and also include in our final Sudakov-resummed coefficient function terms which, in the  $N$ -space coefficient function, are constant with respect to  $N$  and which we denote with  $K_i(\mu_F^2, m^2, Q^2)$ :

$$H_{N,i}^S(\mu^2, \mu_F^2, m^2, Q^2) = \left[ 1 + \frac{\alpha_S(\mu^2) C_F}{2\pi} K_i(\mu_F^2, m^2, Q^2) \right] \times \exp \left[ \ln N g^{(1)}(\ell) + g^{(2)}(\ell, \mu, \mu_F) \right]. \quad (40)$$

The explicit expression for the constant terms reads:

$$K_i(\mu_F^2, m^2, Q^2) = \left( \frac{3}{2} - 2\gamma_E \right) \ln \left( \frac{Q^2 + m^2}{\mu_F^2} \right) + \ln(1 - \lambda) \left( 2\gamma_E + \frac{3\lambda - 2}{2\lambda} \right) + A_i - 2\text{Li}_2 \left( -\frac{Q^2}{m^2} \right) + 2(\gamma_E - 1)(\gamma_E + 2), \quad (41)$$

where  $\lambda$  was introduced in Eq. (8) and we have defined the quantities  $A_i$  [20], which are given by:

$$A_1 = A_3 = 0, \quad (42)$$

$$A_2 = \frac{1}{\lambda} (1 - \lambda) \ln(1 - \lambda). \quad (43)$$

We now match the resummed coefficient function to the exact first-order result: we add the resummed result to the exact coefficient function and, in order to avoid double counting, we subtract what they have in common. Our final result for the resummed coefficient function reads:

$$H_{N,i}^{\text{res}}(\mu^2, \mu_F^2, m^2, Q^2) = H_{N,i}^S(\mu^2, \mu_F^2, m^2, Q^2) - \left[ H_{N,i}(\mu^2, \mu_F^2, m^2, Q^2) \right]_{\alpha_S} + C_{N,i}^q(\mu^2, \mu_F^2, m^2, Q^2) \quad (44)$$

where  $C_{N,i}^q$  is the Mellin transform of the exact  $\mathcal{O}(\alpha_S)$  coefficient functions  $C_i^q$  in Eq. (18).

Before closing this section, we would like to compare our calculation with the results on soft resummation in DIS processes when the final-state quark  $q_2$  is light [8, 10, 11]. In fact, while for top decay, up to width effects, the value of  $m_t^2/m_W^2$  is fixed and rather large,  $m^2/Q^2$  can vary widely in CC DIS according to the value of  $Q^2$  in the considered process. This implies that, if  $m^2 \ll Q^2$ , the heavy quark behaves in a massless fashion and one may have to use a different result for the resummed coefficient function.

As we have already remarked in Section 2, one cannot just recover the massless limit from the massive result: the limits  $m/Q \rightarrow 0$  or  $\lambda \rightarrow 1$  of Eqs. (27) and (40) appear

in fact to be divergent. The actual result is instead finite: one cannot naively apply those equations for very small values of  $m/Q$ , but rather needs to perform a different calculation in this limit.

In fact, if the final-state  $q_2$  is massless, the NLL resummed coefficient function receives an additional contribution due to purely collinear radiation. As shown in [8], one can account for such a contribution by solving a modified evolution equation for the parton distribution associated with the final state.

Therefore, in the massless approximation, one will have to add an additional term to  $S(\alpha_S)$  to account for collinear-enhanced radiation, which leads to the introduction in the Sudakov exponent of a function that is usually denoted by  $1/2B[\alpha_S(Q^2(1-z))]$ . The argument of function  $B(\alpha_S)$  is indeed the virtuality of the final-state jet, which, for a massless  $q_2$  in process (2), will vanish in soft limit like  $(p_2 + p_g)^2 \simeq (1-z)Q^2$ .

On the contrary, if the final-state quark is massive, the virtuality of the final state in the large- $z$  limit is given by  $(p_2 + p_g)^2 = (1-z)(Q^2 + m^2) + m^2$  and is never small, provided  $m/Q$  is sufficiently large. We do not have therefore additional collinear enhancement for heavy quark production, as we do not have any  $\sim B(\alpha_S)$  contribution in the resummed top decay coefficient function of Ref. [17].

Since in the following section we shall present results in the limit  $m/Q \rightarrow 0$  as well, we explicitly write the resummed coefficient function in the massless approximation [10]. As for the large- $m/Q$  case, we write it as an integral over  $z$  and  $k^2 = (p_1 + p_g)^2(1-z)$ , with  $\mu_F^2 \leq k^2 \leq Q^2(1-z)$ :

$$\ln \Delta_N|_{m/Q \rightarrow 0} = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{\mu_F^2}^{Q^2(1-z)} \frac{dk^2}{k^2} A[\alpha_S(k^2)] + \frac{1}{2} B[\alpha_S(Q^2(1-z))] \right\}. \quad (45)$$

The upper limit for the integration variable  $k^2$  can be obtained using Eq. (28): for  $m = 0$  and  $z \rightarrow 1$ , one has  $2E_1 \simeq Q/\sqrt{1-z}$ . From  $k^2 \leq 4E_1^2(1-z)^2$ , the relation  $k^2 \leq Q^2(1-z)$  follows.

We can expand function  $B(\alpha_S)$  as a series in  $\alpha_S$ :

$$B(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n B^{(n)} \quad (46)$$

and, to NLL level, keep only the first term of the expansion [8]:

$$B^{(1)} = -\frac{3}{2} C_F. \quad (47)$$

Explicit results for Eq. (45) to NLL level can be found in Ref. [11], where functions

analogous to our  $g^{(1)}$  and  $g^{(2)}$  are reported <sup>2</sup>. One can also check that the  $\mathcal{O}(\alpha_S)$  expansion of Eq. (45) yields the large- $N$  limit of the NLO coefficient function presented in Eq. (23).

As in Eq. (40), we also include in the final Sudakov-resummed massless coefficient function terms which are constant with respect to  $N$ . Once again, we observe that such constant terms cannot be obtained just taking the  $m/Q \rightarrow 0$  limit of the massive case, i.e. Eq. (41), but we shall have to explicitly extract them from the massless NLO coefficient function. We obtain:

$$K_i(\mu_F^2, m^2, Q^2)|_{m/Q \rightarrow 0} = \left(\frac{3}{2} - 2\gamma_E\right) \ln\left(\frac{Q^2}{\mu_F^2}\right) + \gamma_E^2 + \frac{3}{2}\gamma_E - \frac{\pi^2}{6} - \frac{9}{2}, \quad (48)$$

for  $i = 1, 2, 3$ . As in Eq. (44) we match the resummed result (45) to the exact coefficient function (18), with functions  $H_i^q$  now evaluated in the limit  $m/Q \rightarrow 0$ , i.e.  $\lambda \rightarrow 1$ .

More details on the derivation of Eq. (27) as well as on the comparison with the massless result (45) can be found in [27].

## 4 Phenomenological results

We would like to present results on proton structure functions for charm quark production in CC DIS and investigate the effects of soft-gluon resummation. As discussed in Section 2, structure functions are given by convolutions of  $\overline{\text{MS}}$  coefficient functions with parton distribution functions. Since the resummed coefficient function is given in  $N$ -space, in principle one may like to use parton distribution functions in Mellin space as well, in order to get the resummed structure function in moment space and finally invert it numerically from  $N$ - to  $x$ -space. However, all modern sets of parton distribution functions [28, 29, 30] are given numerically in the form of grids in the  $(x, Q^2)$  space.

In order to overcome this problem, we follow the method proposed in Ref. [25] in the context of joint resummation, which allows one to use  $x$ -space parton distributions, even when performing resummed calculations in Mellin space.

Such a method consists of rewriting the integral of the inverse Mellin transform of the resummed structure functions as follows:

$$\mathcal{F}_i^{\text{res}}(x, Q^2) = \frac{1}{2\pi i} \int_{\Gamma_N} dN \chi^{-N} q_N(\mu_F^2) H_{N,i}^{\text{res}}(\mu^2, \mu_F^2, m^2, Q^2)$$

---

<sup>2</sup>We point out that Ref. [11] contains results to next-to-next-to-leading logarithmic (NNLL) accuracy for soft resummation in massless DIS. However, for the sake of comparison with the massive result (27), we shall use in the following the results of [11] to NLL.

$$= \frac{1}{2\pi i} \int_{\Gamma_N} dN \chi^{-N} [(N-1)^2 q_N(\mu_F^2)] \frac{H_{N,i}^{\text{res}}(\mu^2, \mu_F^2, m^2, Q^2)}{(N-1)^2}, \quad (49)$$

where  $q_N(\mu_F^2)$  is the parton distribution of the initial-state quark  $q_1$ , as if it were available in  $N$  space, and  $\Gamma_N$  is the integration contour in the complex plane, chosen according to the Minimal Prescription [26].

One can integrate out the term with the parton distribution function by observing that the following relation holds:

$$\frac{1}{2\pi i} \int_{\Gamma_N} dN \xi^{-N} (N-1)^2 q_N(\mu_F^2) = \frac{d}{d\xi} \left\{ \xi \frac{d}{d\xi} [\xi q(\xi, \mu_F^2)] \right\} = \Phi(\xi, \mu_F^2). \quad (50)$$

Due to the above relation, one does not need anymore the  $N$ -space parton distributions, but one should just differentiate the  $x$ -space ones, which numerically can be done.

As one has the analytical expression for  $H_{N,i}^{\text{res}}$  in  $N$ -space (44), the following inverse Mellin transform is straightforward:

$$\mathcal{H}(\xi, \mu_F^2) = \frac{1}{2\pi i} \int_{\Gamma_N} dN \xi^{-N} \frac{H_{N,i}^{\text{res}}(\mu^2, \mu_F^2, m^2, Q^2)}{(N-1)^2} \quad (51)$$

which, thanks to the suppressing factor  $1/(N-1)^2$  in the integrand, turns out to be smooth for  $\xi \rightarrow 1$ .

The resummed structure function will be finally expressed as the following convolution:

$$\mathcal{F}_i^{\text{res}}(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} \mathcal{H}(\xi, \mu^2, \mu_F^2) \Phi\left(\frac{x}{\xi}, \mu_F^2\right). \quad (52)$$

After having clarified how to deal with the parton distribution functions, we are able to present results for soft-resummed structure functions. We shall consider charm quark production, i.e.  $q_2 = c$  in Eqs. (1) and (2), since processes with charm quarks in the final state play a role for structure function measurements at NuTeV and HERA experiments.

As for the choice of the parton distribution set, in principle, once data on heavy quark production in CC DIS is available, one should use the NLL coefficient functions when performing the parton distribution global fits and get NLL parton densities as well. For the time being, however, we can just use one of the most-updated NLO sets and convolute it with the fixed-order or resummed coefficient function.

We shall present results based on the new generation of CTEQ NLO  $\overline{\text{MS}}$  parton distribution functions [28], the so-called CTEQ6M set, but similar results can be obtained using, e.g., the MRST [29] or the GRV [30] sets.

The elementary scattering processes which yield the production of charm quarks in CC DIS are  $dW^* \rightarrow c$  and  $sW^* \rightarrow c$ . For  $e^+(e^-)p$  scattering at HERA, our parton distribution function  $q_1(\xi, Q^2)$  in Eq. (17) will be as follows:

$$q_1(\xi, Q^2)|_{\text{HERA}} = |V_{cd}|^2 d(\xi, Q^2) + |V_{cs}|^2 s(\xi, Q^2), \quad (53)$$

where  $V_{cd}$  and  $V_{cs}$  are the relevant Cabibbo–Kobayashi–Maskawa matrix elements. For neutrino scattering on an isoscalar target, Eq. (53) will have to be modified in order to account for the possibility of an interaction with a neutron as well:

$$q_1(\xi, Q^2)|_{\text{NuTeV}} = |V_{cd}|^2 \frac{d(\xi, Q^2) + u(\xi, Q^2)}{2} + |V_{cs}|^2 s(\xi, Q^2), \quad (54)$$

where  $d(\xi, Q^2)$ ,  $u(\xi, Q^2)$  and  $s(\xi, Q^2)$  are still the proton parton distribution functions and we have applied isospin symmetry, i.e.  $u_p = d_n$  and  $s_p = s_n$ .

In order to be consistent with the use of the CTEQ parton distribution functions, we shall use for the QCD scale in the  $\overline{\text{MS}}$  renormalization scheme the values  $\Lambda_4 = 326$  MeV and  $\Lambda_5 = 226$  MeV, for four and five active flavours respectively. This corresponds to  $\alpha_S(m_Z) \simeq 0.118$ . The charm and bottom quark masses have been set to  $m_c = 1.3$  GeV and  $m_b = 4.5$  GeV, as was done in [28].

Since mass effects are important for large values of the ratio  $m_c^2/Q^2$ , we would like to have  $Q$  as small as possible so as to be able to apply our massive resummation. The NuTeV experiment is indeed able to measure structure functions at small  $Q^2$  values [1, 2], as it can detect the final-state muon produced in  $\nu_\mu N \rightarrow \mu X$  processes. In our phenomenological analysis we shall consider charm production at  $Q^2 = 2$  GeV<sup>2</sup> and  $Q^2 = 5$  GeV<sup>2</sup>, which are values reachable at NuTeV such that  $m_c^2/Q^2$  is relatively large.

The detection of CC events at HERA is more problematic, due to backgrounds and to the presence of a neutrino, instead of a charged lepton, in the final state. The current high-luminosity HERA II experiments may detect heavy quarks in CC events, but the  $Q^2$  values for such events are still supposed to be much larger than the charm quark mass squared, typically  $Q^2 \gtrsim 100$  GeV<sup>2</sup> [31]. We shall nonetheless present results for charm quark production at HERA, but in this case we shall have to use the massless result for the resummed  $\overline{\text{MS}}$  coefficient function, i.e. Eq. (45) and the formulas reported in [11]. We shall consider the typical HERA values  $Q^2 = 300$  GeV<sup>2</sup> and 1000 GeV<sup>2</sup>.

We present results for the structure function  $F_2^c(x, Q^2)$ , but we can already anticipate that the effect of the resummation is approximately the same on all three structure functions  $F_1^c$ ,  $F_2^c$  and  $F_3^c$  and on the single-differential cross section  $d\sigma/dx$ , obtained from Eq.(11), after integrating over  $y$ .

For the sake of comparison with the experiments, we shall plot the structure function  $F_2^c$  in terms of the Bjorken  $x$ , which is the measured quantity. For most of our plots,

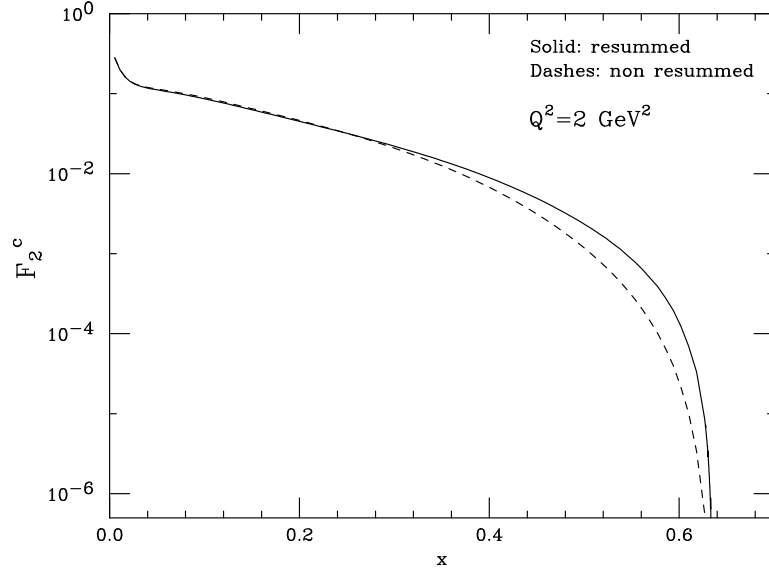


Figure 1: Results for  $F_2^c(x, Q^2)$  for charm quark production in neutrino scattering at  $Q^2 = 2 \text{ GeV}^2$  with (solid) and without (dashed) soft resummation in the coefficient function. We have set  $\mu_F = \mu = Q$ .

we shall set renormalization and factorization scales equal to  $Q$ , i.e.  $\mu_F = \mu = Q$ . Afterwards, we shall also investigate the dependence of our results on the choice of such scales.

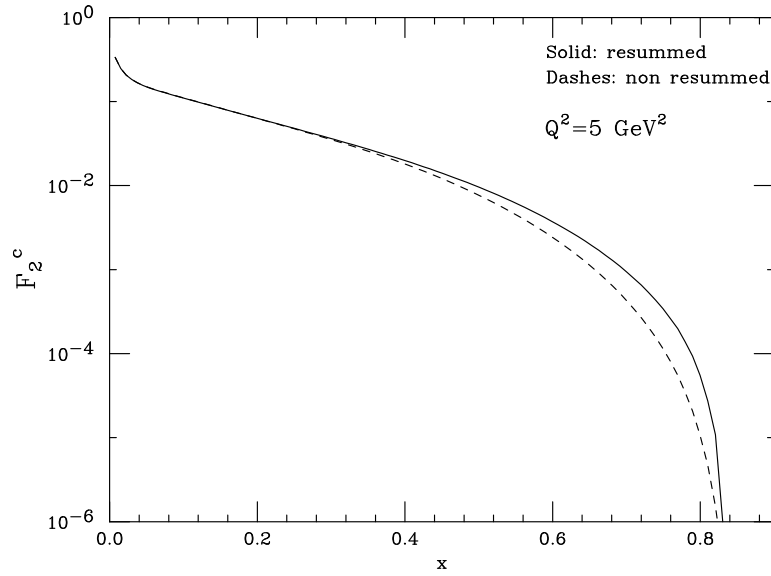


Figure 2: As in Fig. 1, but for  $Q^2 = 5 \text{ GeV}^2$ .

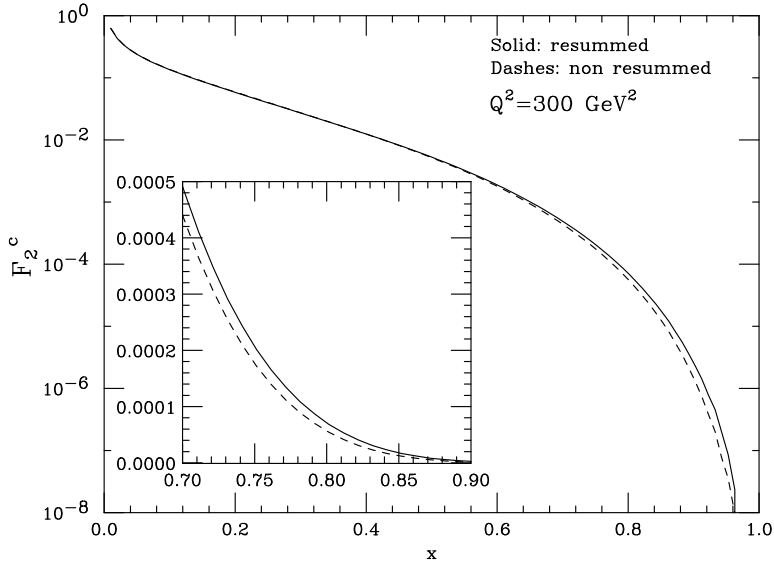


Figure 3: Results for  $F_2^c(x, Q^2)$  for positron-proton scattering at  $Q^2 = 300 \text{ GeV}^2$ , with (solid) and without (dashes) soft resummation in the coefficient function. We have set  $\mu_F = \mu = Q$ . In the inset figure, we show the same plots at large  $x$  and on a linear scale.

In Figs. 1 and 2 we show  $F_2^c(x, Q^2)$  in neutrino scattering at  $Q^2 = 2 \text{ GeV}^2$  and  $5 \text{ GeV}^2$  and compare the results obtained using fixed-order and soft-resummed  $\overline{\text{MS}}$  coefficient functions. We have set the target mass to  $M = 1 \text{ GeV}$ , which is a characteristic nucleon mass. We observe a relevant effect of the implementation of soft resummation: the two predictions agree up to  $x \simeq 0.2 - 0.3$ , afterwards one can see an enhancement of the structure function due to the resummation. At very large  $x$ , resummed effects are indeed remarkable: for  $Q^2 = 2 \text{ GeV}^2$  one has an enhancement of a factor of 2 at  $x = 0.5$  and a factor of 5 at  $x = 0.6$ . For  $Q^2 = 5 \text{ GeV}^2$  one gets a factor of 2 at  $x = 0.7$  and 5 at  $x = 0.8$ .

In Figs. 3 and 4 we present results for  $F_2^c(x, Q^2)$ , but for charm production at HERA, in particular for positron-proton scattering at  $Q^2 = 300$  and  $1000 \text{ GeV}^2$ . In this case, since  $m_c/Q \ll 1$ , we use the massless result (45) for the resummed coefficient function. We see that the impact of the resummation is smaller than in the case of low  $Q^2$  values. This is a reasonable result: in fact, leading and next-to-leading logarithms in the Sudakov exponent are weighted by powers of  $\alpha_S(\mu^2)$ . As, for example,  $\alpha_S(2 \text{ GeV}^2) \simeq 3 \alpha_S(300 \text{ GeV}^2)$ , resummed effects are clearly more important when  $Q^2$  is small. Moreover, the larger  $Q^2$  is, the larger the values of  $x$  are at which one is sensitive to Sudakov effects.

We note in Figs. (3) and (4) that, for  $x > 0.6$ , fixed-order and resummed predictions start to be distinguishable. We estimate the overall impact of large- $x$  resummation on



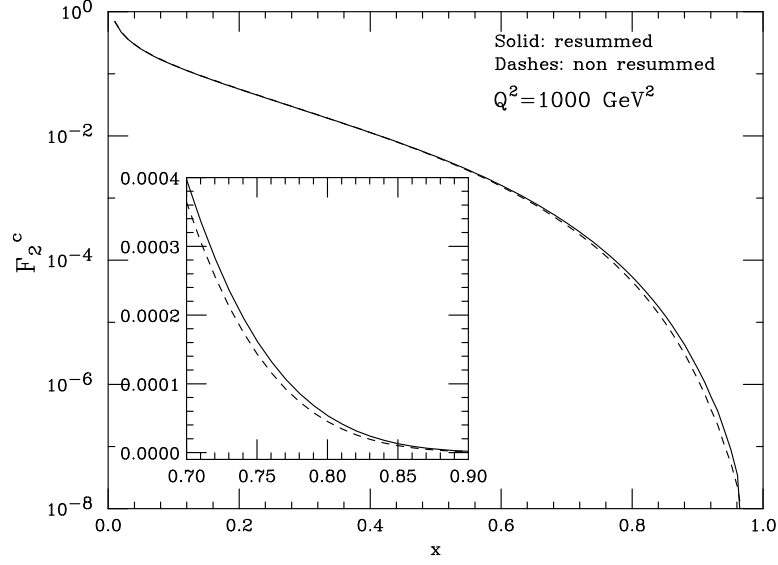


Figure 4: As in Fig. 3, but for  $Q^2 = 1000 \text{ GeV}^2$ .

$F_2^c(x, Q^2)$  at  $Q^2 = 300 \text{ GeV}^2$  and  $Q^2 = 1000 \text{ GeV}^2$  to be between 10% and 20%.

In Figs. 5 and 6 we investigate the dependence on factorization and renormalization scales. We still keep  $\mu = \mu_F$ , but we allow such scales to assume the values  $Q^2$ ,  $2Q^2$  and

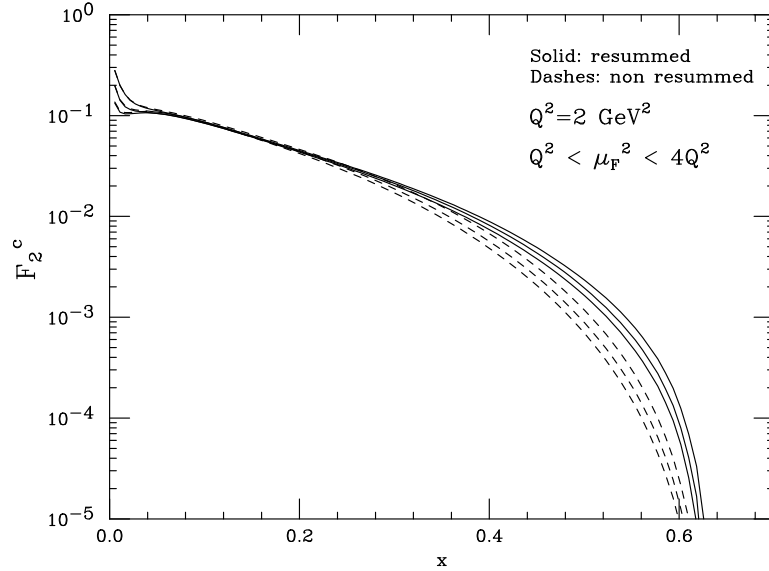


Figure 5: Dependence of  $F_2^c$  on the factorization scale for neutrino scattering at  $Q^2 = 2 \text{ GeV}^2$ . Solid lines include soft resummation in the coefficient function, dashed lines are fixed-order predictions.

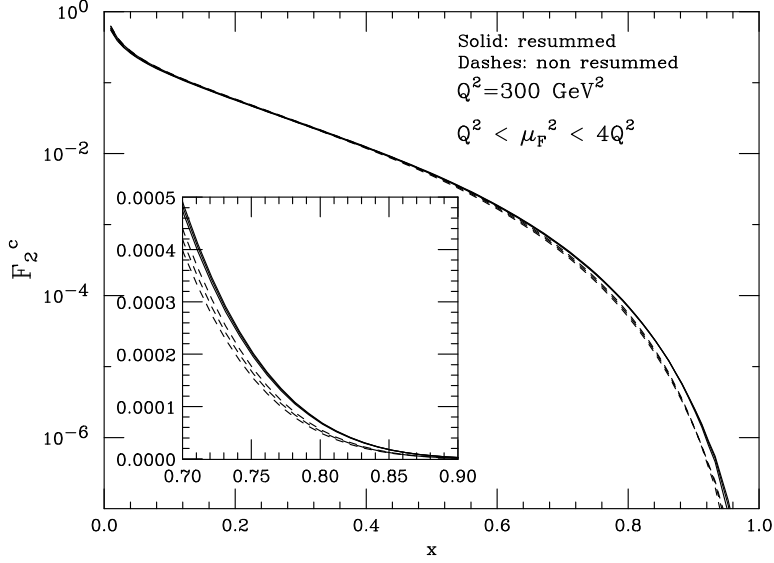


Figure 6: As in Fig. 5, but for  $e^+p$  scattering at  $Q^2 = 300 \text{ GeV}^2$ . In the inset figure, we plot the same curves at large  $x$  and on a linear scale.

$4Q^2$ . We consider  $Q^2$  values of 2 and  $300 \text{ GeV}^2$  for the experimental environments of NuTeV and HERA respectively. We see that the curves which implement soft resummation in the coefficient function show a weaker dependence on the chosen value of the factorization/renormalization scales. In Fig. 5 one can see that one still has a visible effect of the value of such scales, but the overall dependence on  $\mu_F$  and  $\mu$  of the resummed prediction is smaller than for the fixed-order. The plots at large  $Q^2$  exhibit, in general, a weak dependence on  $\mu_F$  and  $\mu$  even at NLO, as shown in Fig. 6: in fact, the dependence on the factorization and renormalization scales is logarithmic, and hence smaller once  $\mu$  and  $\mu_F$  vary around large values of  $Q$ . Nevertheless, while the NLO structure function still presents a residual dependence on the scales, the resummed predictions obtained with three different values of  $\mu_F$  are basically indistinguishable. A smaller dependence on such scales implies a reduction of the theoretical uncertainty of the prediction and is therefore a remarkable effect of the implementation of soft gluon resummation.

We finally would like to compare the impact that mass effects and soft-gluon resummation have on the charm structure functions. To achieve this goal, we plot in Fig. 7 the theoretical structure function  $\mathcal{F}_2^c(\chi, Q^2)$  defined in Eq. (17) as a function of the variable  $\chi$  (see Eq. (7)), for neutrino scattering at  $Q^2 = 2 \text{ GeV}^2$ . We compare fixed-order massive (dashed line), fixed-order massless (dots) and massive resummed (solid) predictions. We observe that the two fixed-order calculations yield different predictions throughout the full  $\chi$  range, which is a consequence of the implementation of mass effects; however, at large  $\chi$ , where one starts to be sensitive to the resummation, the impact of soft resummation is competitive with the one of mass contributions.

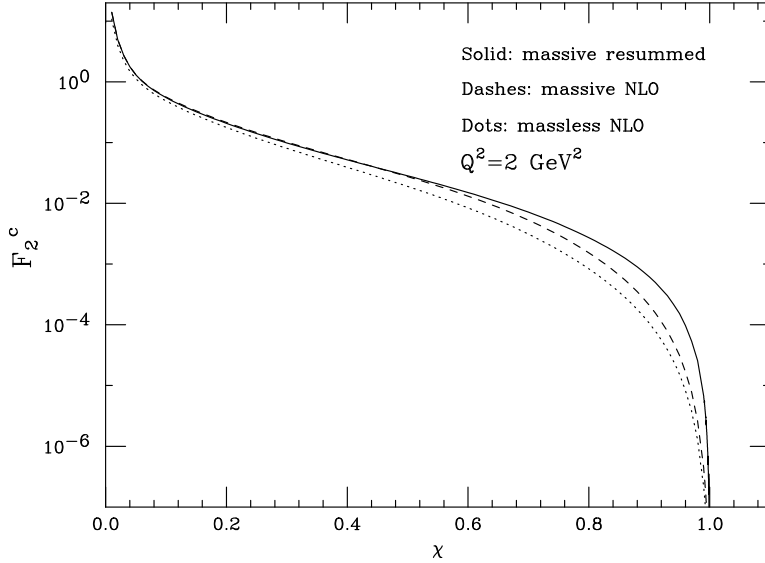


Figure 7: Comparison of massive (dashes) and massless (dots) fixed-order calculations with the resummed massive result (solid) for  $Q^2 = 2 \text{ GeV}^2$  and  $\mu = \mu_F = Q$ . Plotted is the theoretical structure function  $\mathcal{F}_2^c(\chi, Q^2)$  defined in Eq.(17).

Before closing this section, we would like to point out that, although we have improved our perturbative prediction implementing soft-gluon resummation in the coefficient function, non-perturbative corrections are still missing. Non-perturbative effects are important especially at small values of  $Q^2$  and large  $x$ . In fact, a more accurate investigation of the very-large  $x$  limit of the structure functions shows that they exhibit an oscillatory behaviour once  $x$  gets closer to the maximum value which is kinematically allowed.

## 5 Conclusions

We have studied inclusive heavy quark production in charged-current Deep Inelastic Scattering. We have reviewed the results for the NLO cross section and structure functions and observed that the  $\overline{\text{MS}}$  quark-initiated coefficient function contain terms which become arbitrarily large once the initial-state light-quark energy fraction approaches unity, which corresponds to soft-gluon radiation. We have resummed soft-gluon contributions in the  $\overline{\text{MS}}$  coefficient function to next-to-leading logarithmic accuracy. We have compared the results with the resummed coefficient function for  $b$ -quark production in top decay and for CC DIS processes, but with a light quark in the final state.

We have presented results on the structure function  $F_2^c(x, Q^2)$  for charm quark pro-

duction at NuTeV and HERA, using the massive or the massless coefficient function according to the value of the ratio  $m_c/Q$ . We have compared predictions obtained considering fixed-order and resummed coefficient functions and have found a remarkable effect due to soft resummation. The structure functions exhibit an enhancement at large  $x$ , which is visible especially when  $Q$  is comparable with the heavy quark mass. The results including soft resummation exhibit very little dependence on the choice of factorization and renormalization scales, which is a reduction of the theoretical uncertainty. Furthermore, we have compared massive and massless predictions at small  $Q^2$  and have found that, at large  $x$ , resummation effects are of a magnitude similar to mass corrections.

It will be clearly very interesting to compare our predictions with experimental data. New data on NuTeV structure functions will be soon available [32] and we plan to investigate whether the structure functions yielded by our calculation are able to fit the data. In particular, we wish to evaluate the role played by soft-gluon resummation in performing such fits. The preliminary analyses in Ref. [1] show in fact that the NuTeV data lies above the NLO structure functions based on Ref. [33] for  $x \gtrsim 0.55$ . The effect due to the resummation is indeed an enhancement of structure functions at large  $x$ , hence it may throw some light on the NuTeV studies. However, for such a comparison to be reliable, we shall have to implement nuclear-correction effects into our structure functions, to account for the neutrino scattering on an iron target. A similar comparison can be made with the foreseen CC data from HERA II. Moreover, we believe that once such a data becomes available, our calculation can also be used to perform updated NLL global fits of the parton distribution functions.

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